# Simulation-based user equilibrium: improving the fixed point solution methods

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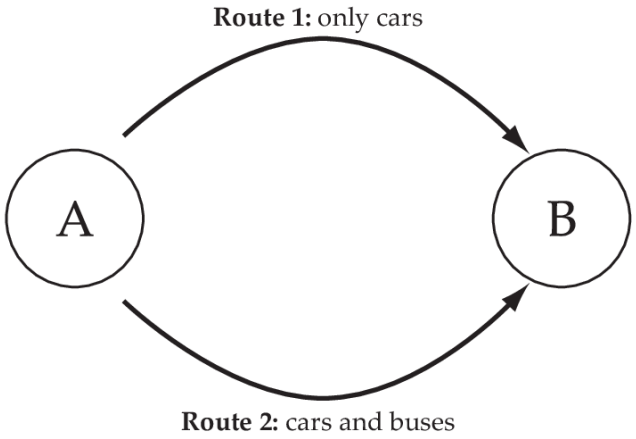
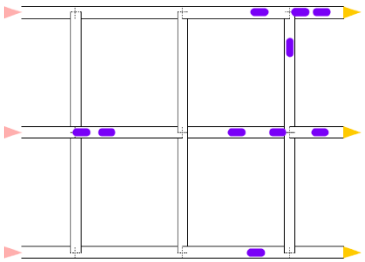
# Introduction (research scope)

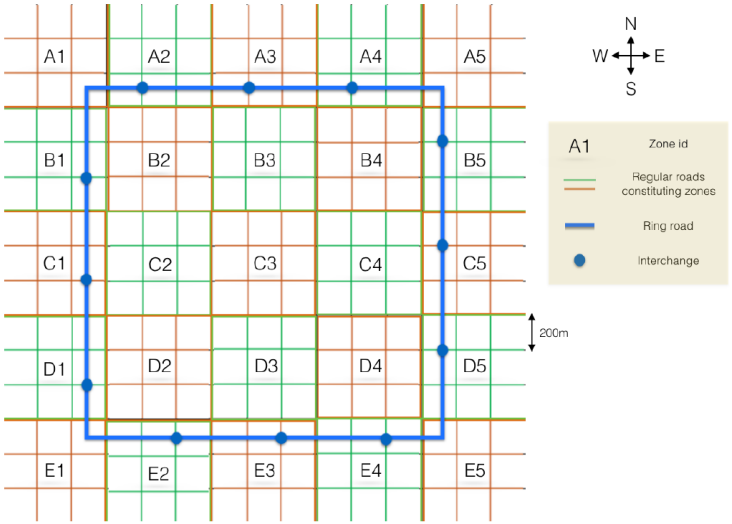
- Traffic assignment problem
  - Input: OD flow
  - Output: Path flow distribution
  - Goals:
    - User Equilibrium (UE)
- Fixed point problem
- Time (dynamic)
  - Dynamic Traffic Assignment (DTA)
- Cost function (time)
- Departure time
- Demand

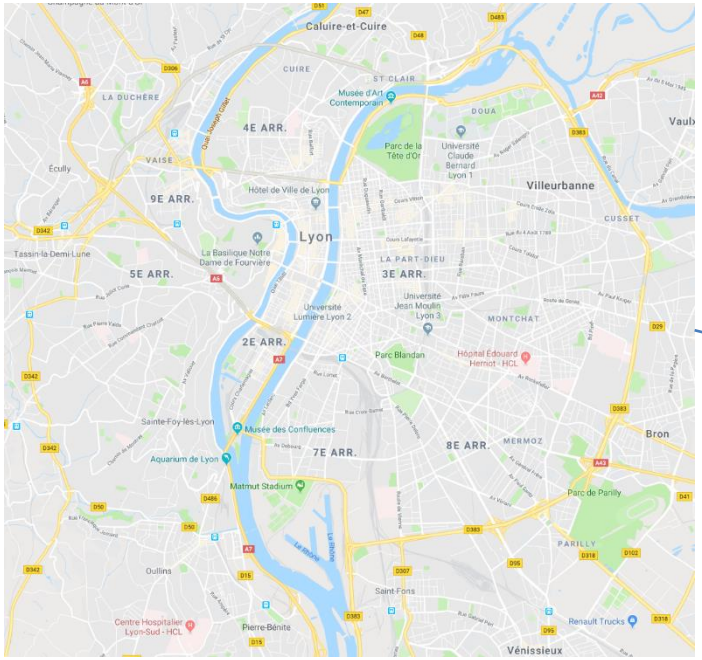



# Introduction (research scope)

- Network Selection



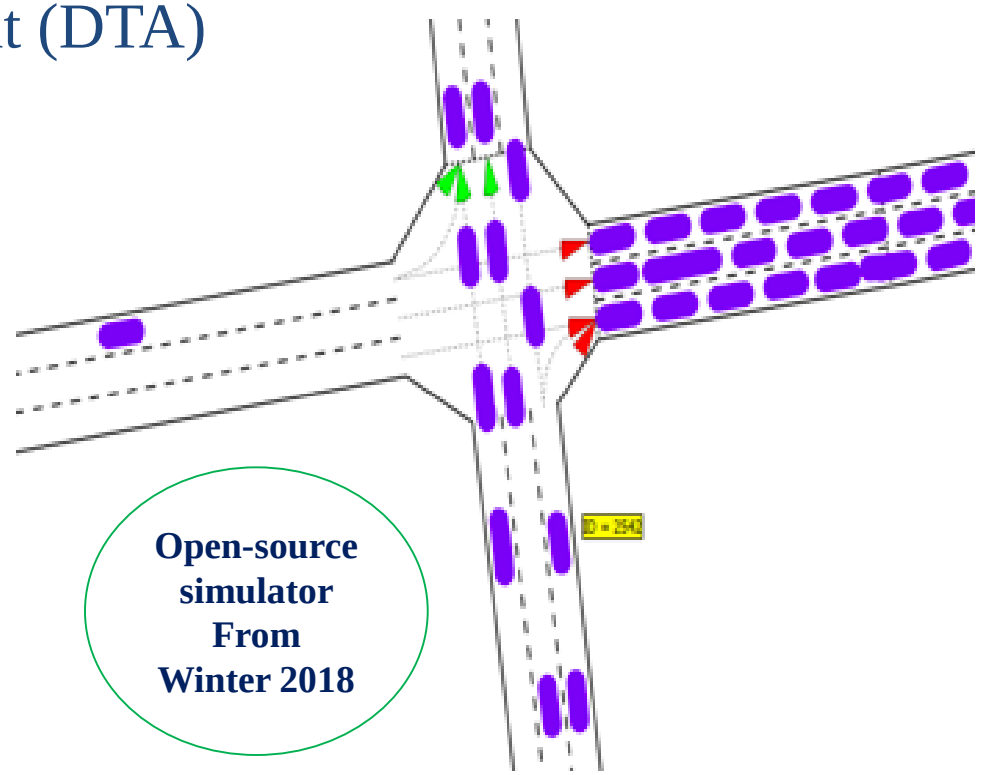



# Introduction (research scope)

Problem Setting:

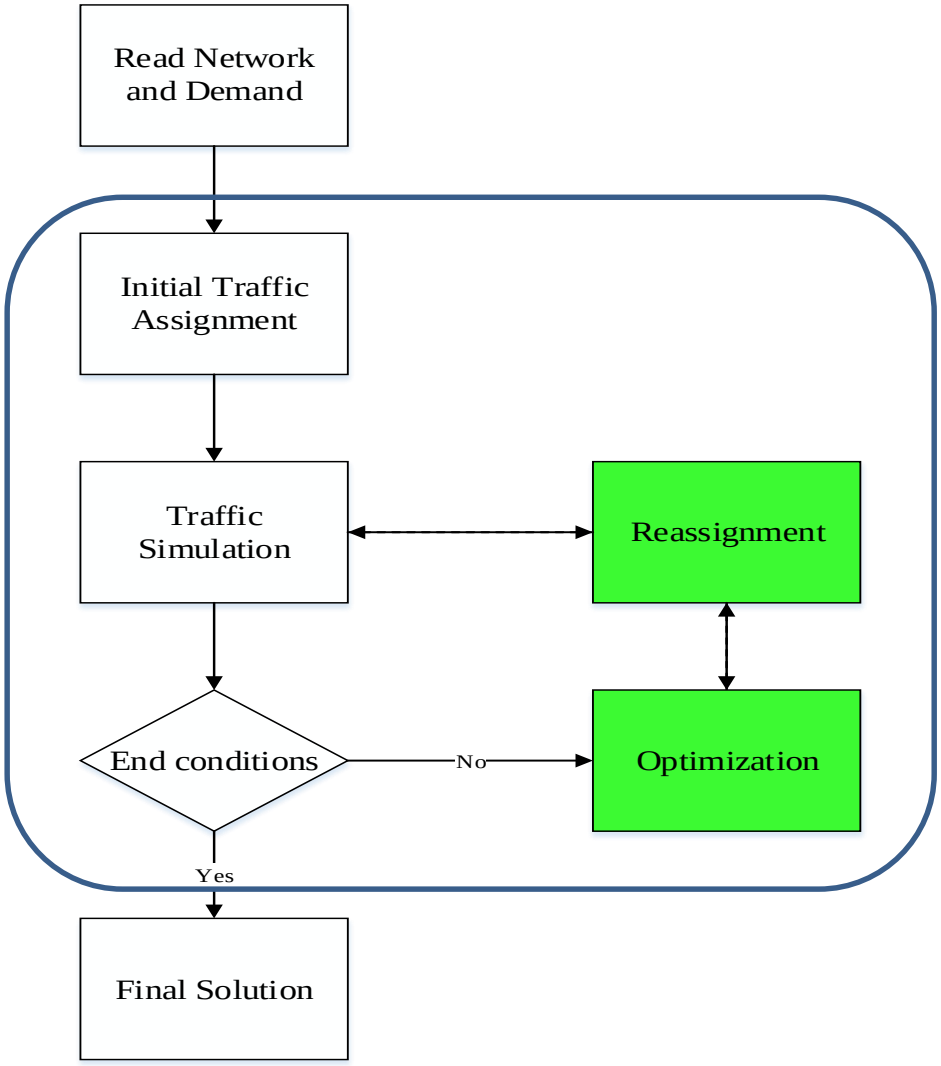
- Simulation-based
- Dynamic Traffic Assignment (DTA)
- Predictive (not reactive)
- Trip-based (not flow-based)
- Link level information
- Mono modal (Unicity)
- Large-scale network
- Time-dependent



**Open-source simulator From Winter 2018**

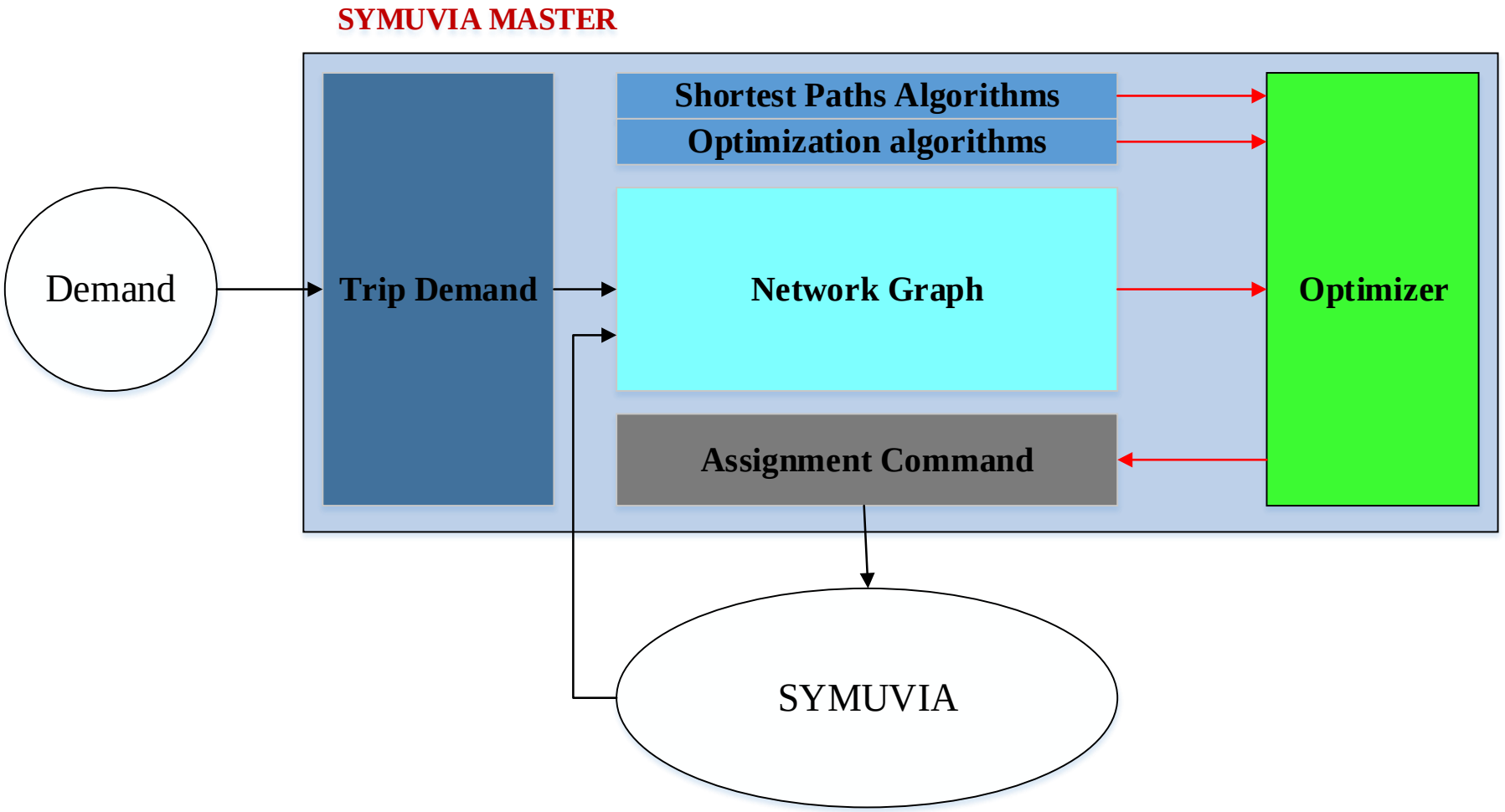
# Dynamic Traffic Assignment (DTA)

## Simulation-based optimization



# Dynamic Traffic Assignment (DTA)

## Simulation-based optimization



# Solution quality

## Multimodal Large-scale network:


Quality indicator

- Average Gap per user [minute]

$$Gap(\mathrm{n}, TT^*) = \frac{\sum_{w\in W}\sum_{\tau=1}^{T}\sum_{p\in P(w,\tau)} n_{w,p,\tau}^{i}\left[TT_{w,p,\tau}^{i} - TT_{w,p,\tau}^{i*}\right]}{\sum_{w\in W}\sum_{\tau=1}^{T}\sum_{p\in P(w,\tau)} n_{w,p,\tau}^{i}}$$

- Violation [%]
  - The user violation: If the gap between user perceive travel time and shortest path travel time is bigger than 10% of the shortest path travel time, the user is in violation.
  - The OD violation: The OD pair $w$ is in violation when there are more than 10% of the users on $w$ are in violation.
  - The violation indicator of network is the share of ODs which are in violation.

# Fast heuristic methods to determine the UE

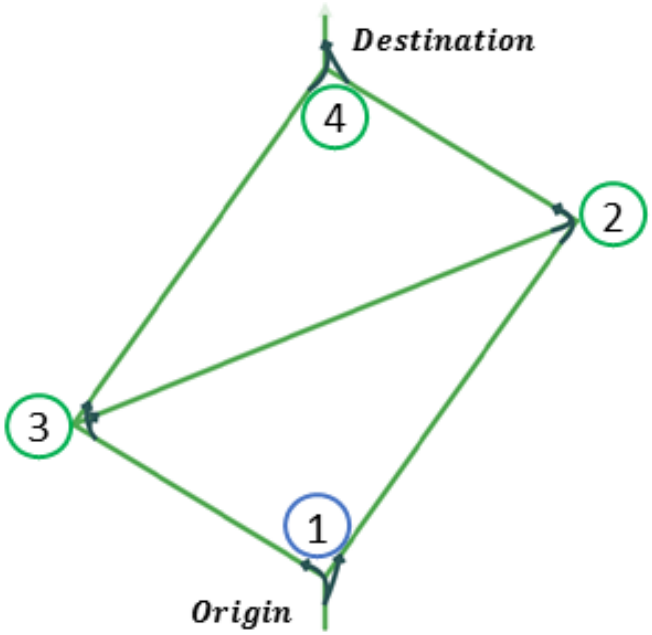
**Scientific Question:**

How can we find the DTA solution with good quality in terms of optimality and feasible computation time (convergence speed)?

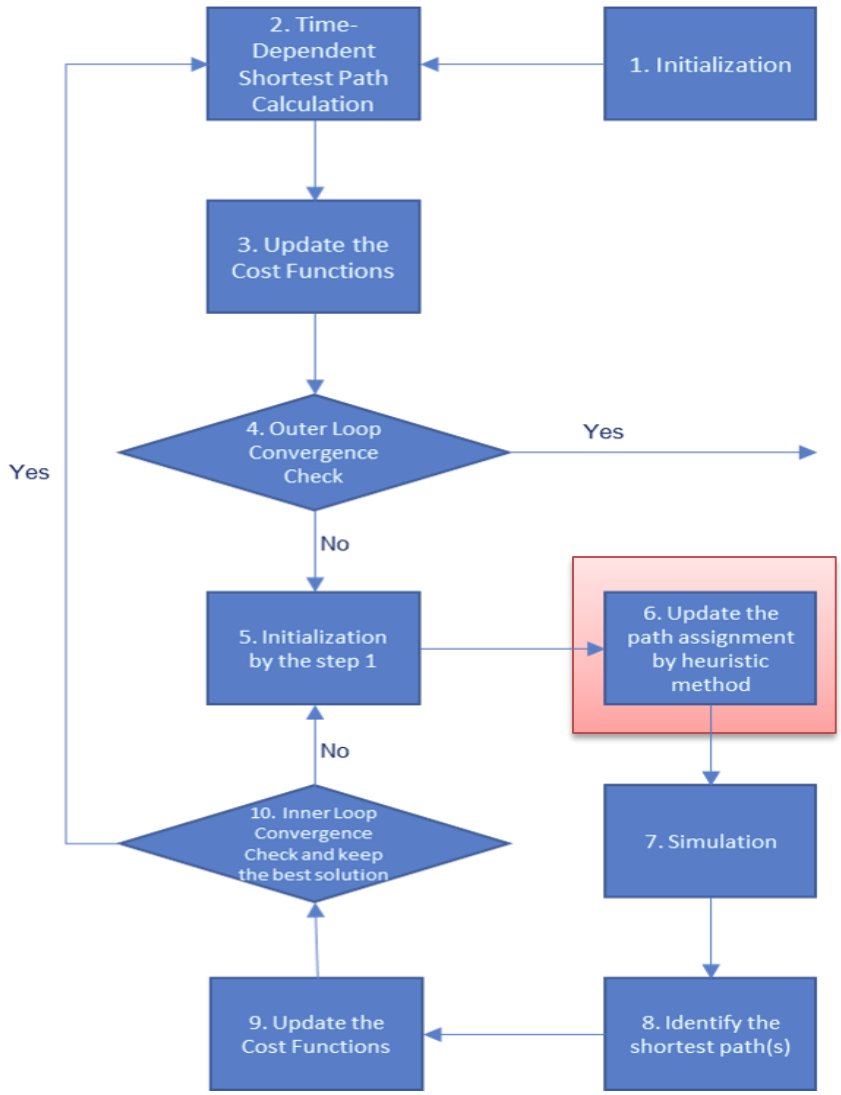
# Fast heuristic methods to determine the UE

## Challenges:

1. Running the shortest path algorithm between all Origin-Destination (OD) pairs in a transportation network.
2. Determining the flow distribution on these paths considering the OD flow demand and the dynamic traffic states inside the network.

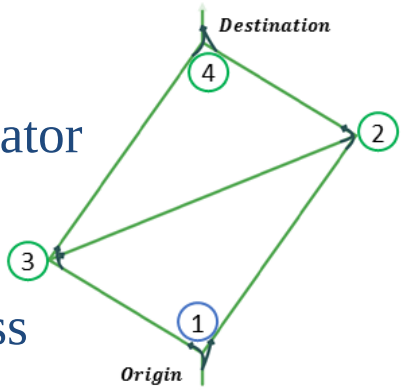


# Equilibration process




## Change the number of users on each :

- Outer loop
  - Path discovery
  - Global quality indicator
- Inner loop
  - Fixed path set
  - Optimization process




- Fixed point algorithms:
  - Classic MSA **[Robbins and Monro, 1951]**
    - Step size: $\sigma_{MSA}^{i} = \frac{1}{i}$
  - MSA Ranking **[Sbayti et al., 2007]**

- Probabilistic

Probability of changing path = $\frac{GC_p - GC_p^*}{GC_p}$

Use random number or class indicator to take decision

# Equilibration process

## Fixed point algorithms:

- **Method of Successive Average (MSA) [Robbins and Monro, 1951]** $\sigma^i_{MSA} = \frac{1}{i}$
- **MSA Ranking [Sbayti et al., 2007]** $\sigma^i_{MSA\ ranking} = \frac{1}{i}$
- **Gap-based method [Lu et al., 2009]** $\sigma^i_{Gap-based} = \frac{1}{i} . \frac{C_p - C_p^*}{C_p}$
- **Hybrid 1 [Halat et al., 2016]** Probability of changing path = $\frac{1}{i} . \frac{C_p - C_p^*}{C_p}$
- **Hybrid 2 [Verbas et al., 2015]** $\sigma^i_{Gap-based} = \frac{1}{i} . \frac{C_p - C_p^*}{C_p}$ Choose users by Prob. method
- **Probabilistic method [Ameli et al., 2017]** **Free from step size**
- **Hybrid 3:**

$$NS^p_{Hybrid} = min \left\{ n(Tr_p),\ [\sigma^i_{Gap-based} . n(Tr_p)] . \frac{\sigma^i_{Gap-based}}{\sigma^i_{MSA}} \right\}$$

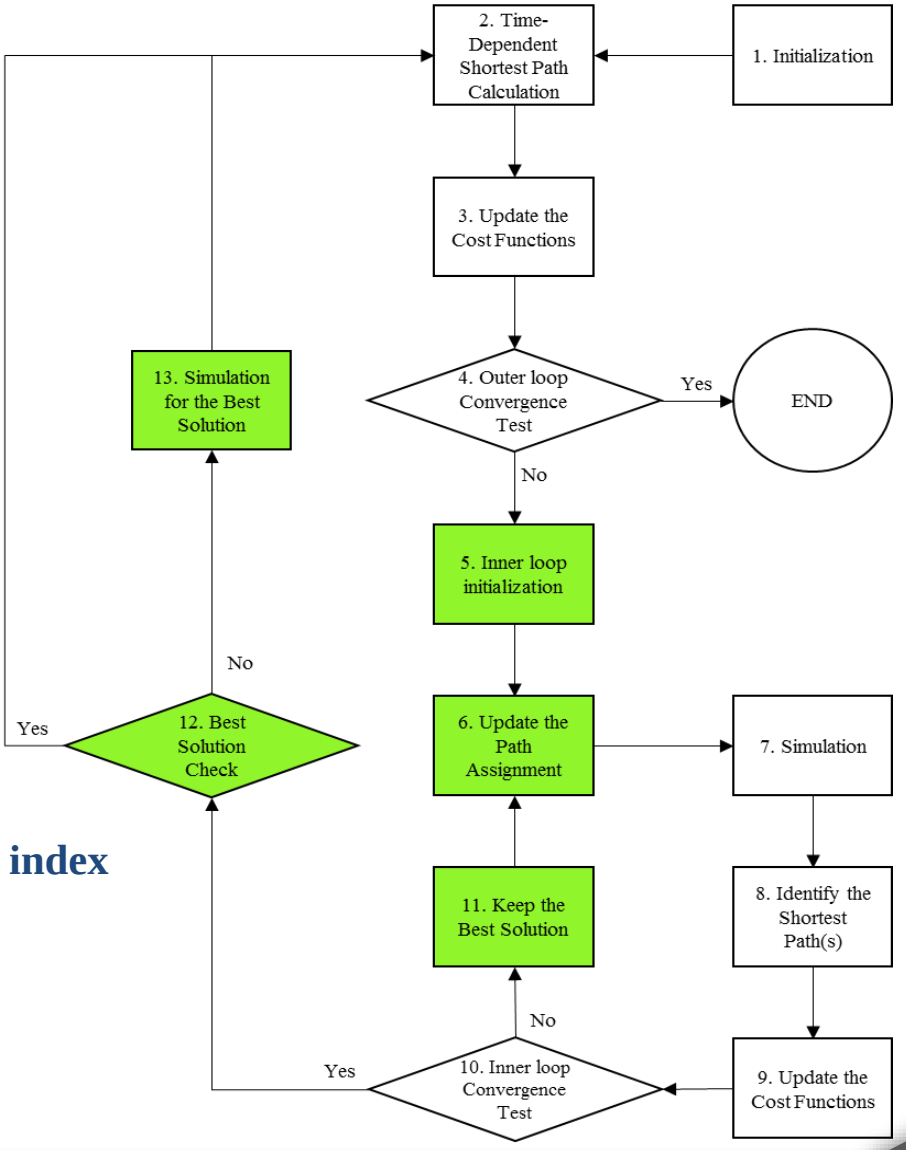
- **Gap-based normalized:**

$$\sigma^i_{Gap-based\ normalized} = \frac{\hat{C}_{p,\tau} - \hat{C}^*_{w,\tau}}{\sum_{p \in P(w,\tau)} (\hat{C}_{p,\tau} - \hat{C}^*_{w,\tau})} . \sigma^i_{MSA}$$

# Equilibration process

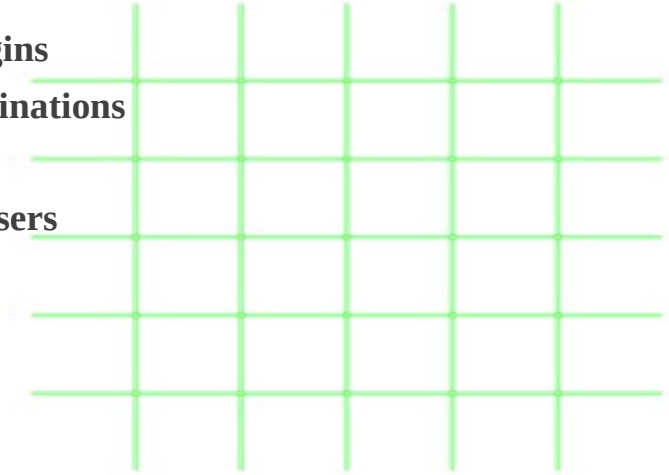
## Improvements:

- Keep the best solution for each outer loop
- Benchmark different algorithms
- Inner loop initialization
  - 1- All-or-nothing
  - 2- Uniform initialization
  - 3- Keep the assignment pattern
- Initial step size selection
  - 1- Reinitializing the step size by inner loop index
  - 2- Smart step size

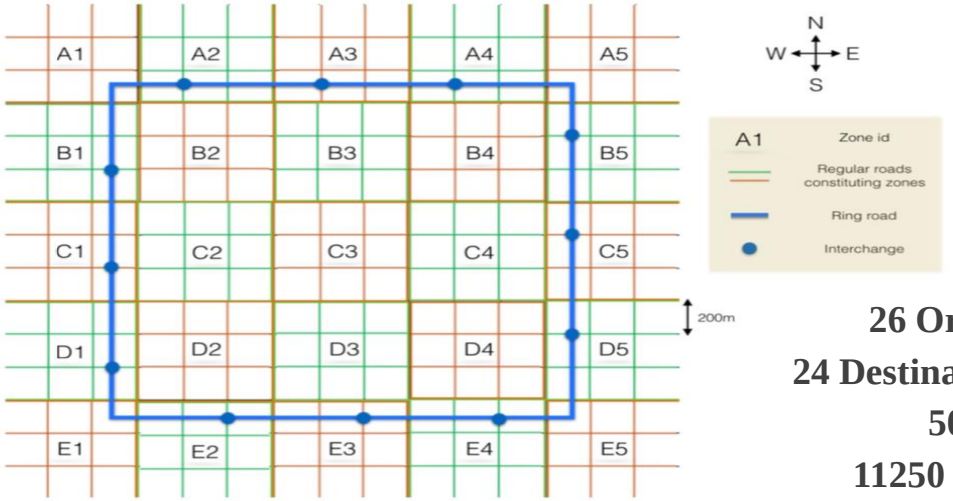


# Test cases

19 Origins
16 Destinations
2 hours
5,202 users

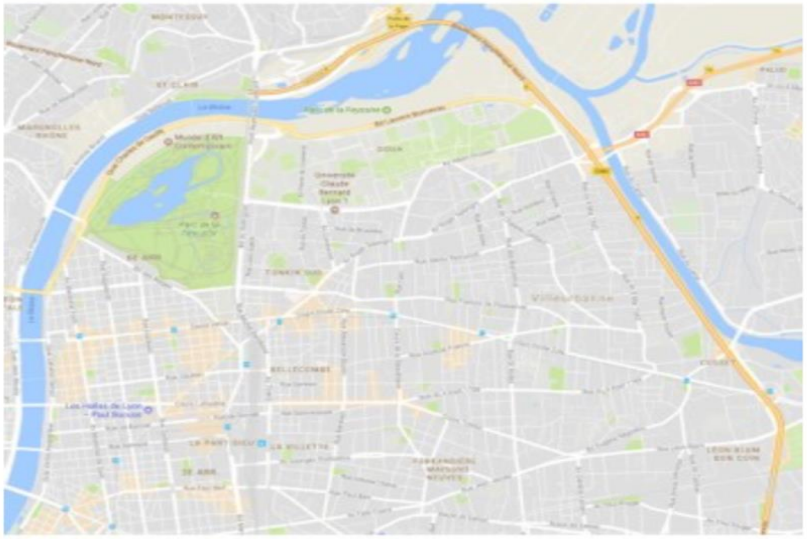


(a) 5 × 5 grid network




(b) Manhattan-like network with ring road

26 Origins
24 Destinations
50 min
11250 users

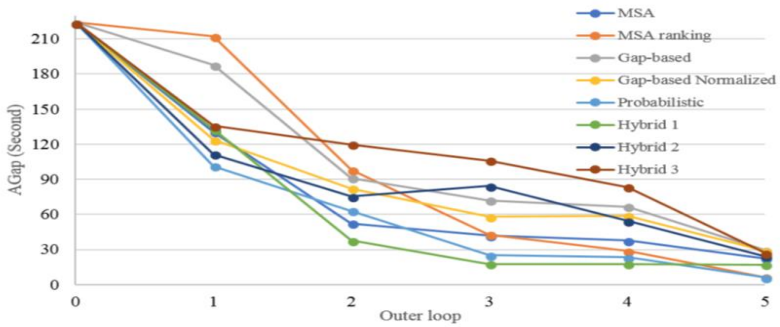


(c) Mapping data ©Google 2018

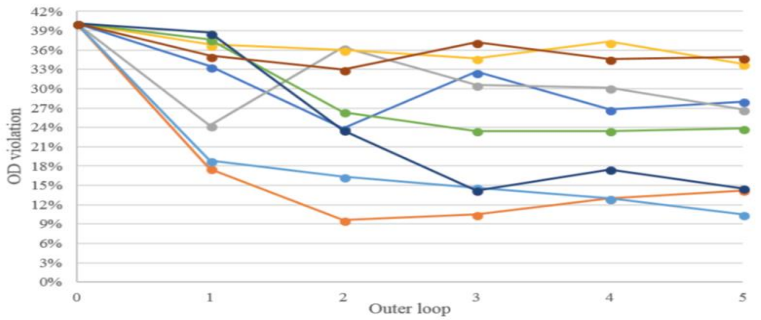
(d) The traffic network using by Symuvia

1,883 Nodes
5,935 Links
94 Origins
227 Destinations
2.5 hours
54190 users

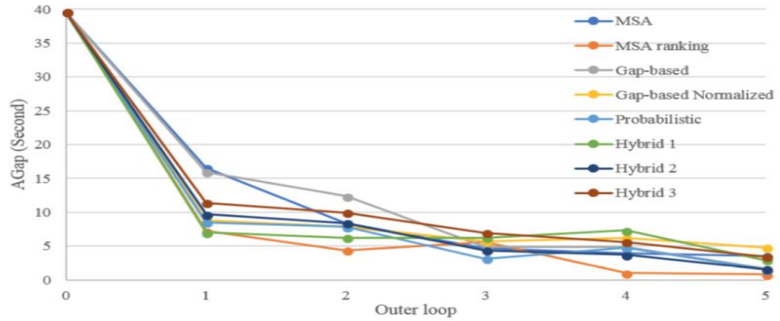
# Numerical results (swap formulas)



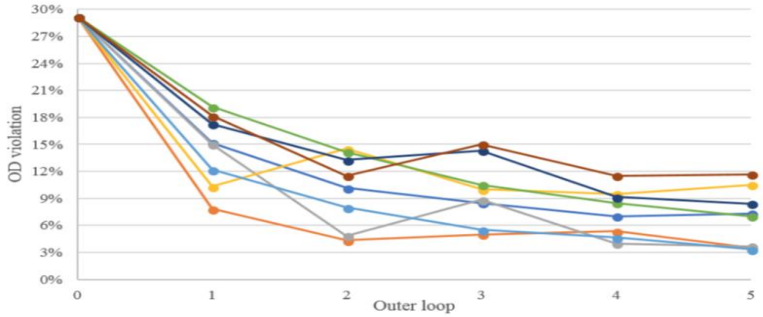

(a) AGap indicator for 5by5 grid



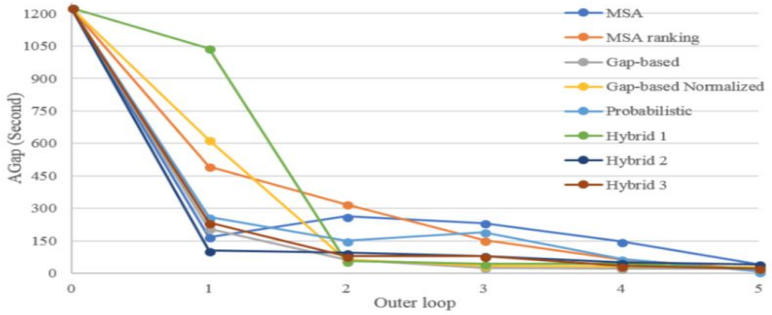

(b) Violation indicator for 5by5 grid



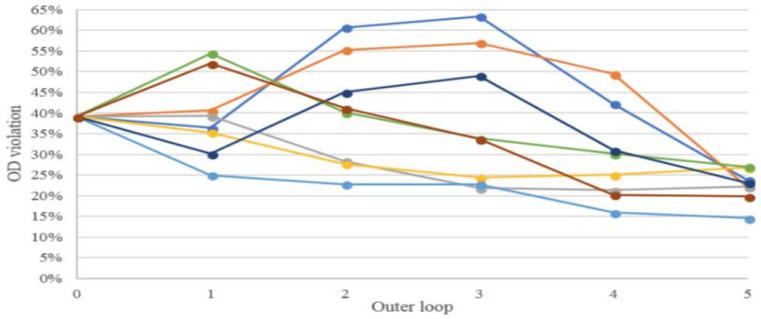

(c) AGap indicator for Ring city




(d) Violation indicator for Ring city




(e) AGap indicator for Lyon6V




(f) Violation indicator for Lyon6V

# Numerical results (Convergence patterns for the swap formulas)

TABLE 1 Results of swap methods for three networks [*AGap* (second)]

| Swap formula / Network | 5by5 | | Ring city | | Lyon6V | |
|---|---|---|---|---|---|---|
| | AGap | Violation | AGap | Violation | AGap | Violation |
| MSA | 22.33 | 0.28 | 3.56 | 0.07 | 40.72 | 0.24 |
| MSA ranking | 6.34 | 0.14 | 0.82 | 0.04 | 19.25 | 0.22 |
| Gap-based | 29.32 | 0.27 | 1.62 | 0.04 | 21.65 | 0.22 |
| Gap-based Normalized | 28.74 | 0.34 | 4.79 | 0.11 | 29.48 | 0.27 |
| Probabilistic | 5.92 | 0.10 | 1.55 | 0.03 | 7.42 | 0.15 |
| Hybrid 1 | 16.93 | 0.24 | 2.96 | 0.07 | 22.96 | 0.27 |
| Hybrid 2 | 24.25 | 0.15 | 1.60 | 0.08 | 41.25 | 0.23 |
| Hybrid 3 | 26.92 | 0.35 | 3.44 | 0.12 | 22.68 | 0.20 |

Probabilistic method works better than others methods in all networks.

# Numerical results (Inner loop initialization)

TABLE 2 Results of initialization methods [*AGap* (second)]

| Scenario / Network | | 5by5 | | Ring City | | Lyon6V | |
|---|---|---|---|---|---|---|---|
| Swap formula | Method | AGap | Violation | AGap | Violation | AGap | Violation |
| MSA | All-or-nothing | 22.33 | 0.28 | 3.56 | 0.07 | 40.72 | 0.24 |
| | Uniform | 18.51 | 0.31 | 2.90 | 0.09 | 41.25 | 0.31 |
| | Keep solution | 18.99 | 0.38 | 1.89 | 0.10 | 23.24 | 0.24 |
| MSA Ranking | All-or-nothing | 6.34 | 0.14 | 0.82 | 0.04 | 19.25 | 0.22 |
| | Uniform | 15.91 | 0.23 | 4.94 | 0.08 | 17.18 | 0.22 |
| | Keep solution | 13.13 | 0.13 | 3.40 | 0.05 | 8.81 | 0.18 |
| Gap-based | All-or-nothing | 29.32 | 0.27 | 1.62 | 0.04 | 21.65 | 0.22 |
| | Uniform | 24.40 | 0.24 | 6.70 | 0.21 | 30.45 | 0.22 |
| | Keep solution | 21.62 | 0.23 | 5.87 | 0.22 | 20.53 | 0.26 |
| Probabilistic | All-or-nothing | 5.92 | 0.10 | 1.55 | 0.03 | 7.42 | 0.15 |
| | Uniform | 8.87 | 0.13 | 4.27 | 0.11 | 55.11 | 0.29 |
| | Keep solution | 4.49 | 0.10 | 1.06 | 0.04 | 7.01 | 0.13 |

Keeping the assignment improves the results in the large-scale network.

# Numerical results (step size selection)

TABLE 3 Results of initial step size methods [*AGap* (second)]

| Scenario / Network | | 5by5 | | Ring City | | Lyon6V | |
|---|---|---|---|---|---|---|---|
| Swap formula | Method | AGap | Violation | AGap | Violation | AGap | Violation |
| MSA ranking and Keep solution | Initial | 13.13 | 0.13 | 3.40 | 0.05 | 8.81 | 0.18 |
| | Reset | 20.71 | 0.29 | 3.22 | 0.09 | 8.67 | 0.18 |
| | Smart | 30.91 | 0.37 | 7.83 | 0.10 | 55.89 | 0.24 |
| Gap-based and All-or-nothing | Initial | 29.32 | 0.27 | 1.62 | 0.04 | 19.25 | 0.22 |
| | Rest | 14.53 | 0.23 | 5.08 | 0.12 | 23.23 | 0.25 |
| | Smart | 53.08 | 0.34 | 4.55 | 0.16 | 13.88 | 0.39 |

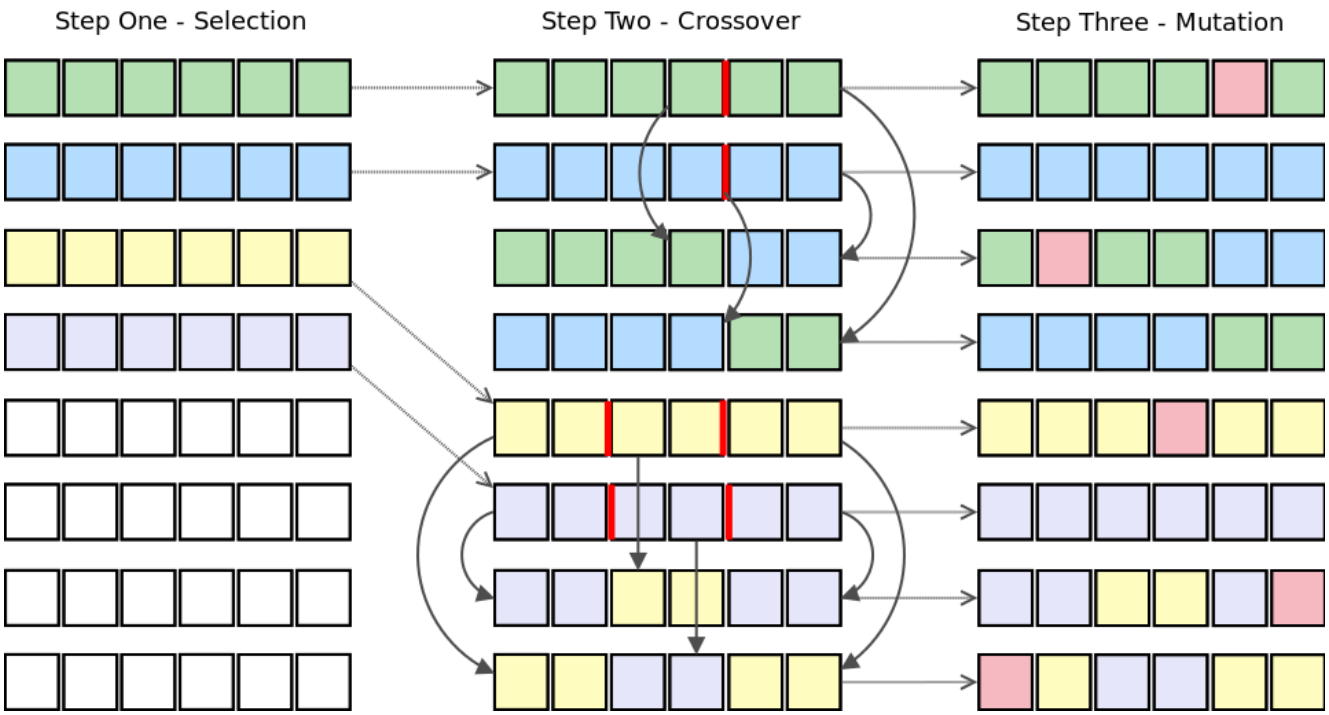
Smart step size works better for Gap-based method the large-scale network.

# Conclusion

- The performance of the optimization methods depend on the network size.
- Improvements to the solution algorithm:
  - Keeping the best assignment pattern during the inner loop iterations
  - Three new swapping methods
  - Two new methods for the initialization of the step size
  - Two alternative methods to initialize the assignment pattern at the beginning of the outer loop.
- In the large-scale network, the combination of Probabilistic approach with keeping the assignment solution of the previous outer loop works better than other methods.

# Future Work

- Apply more methods to different network sizes
- Compare the performance and computation time of various methods
- Use meta-heuristic methods in inner loop



# Thanks for your attention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